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Scannell, Brian; Rippeth, Thomas; Simpson, John; Polton, Jeff; Hopkins, Jo

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**Correcting surface wave bias in structure function estimates of turbulent
kinetic energy dissipation rate**

Brian D. Scannell*, Tom P. Rippeth and John H. Simpson

School of Ocean Sciences, Bangor University, Menai Bridge, Isle of Anglesey, United Kingdom

Jeff A. Polton and Joanne E. Hopkins

National Oceanography Centre, Joseph Proudman Building, Liverpool, United Kingdom

*Corresponding author address: School of Ocean Sciences, Bangor University, Menai Bridge, Isle
of Anglesey, LL59 5AB, United Kingdom.

E-mail: brian.scannell@bangor.ac.uk

ABSTRACT

10 The combination of acoustic Doppler current profilers and the structure
11 function methodology provide an attractive approach to making extended time
12 series measurements of oceanic turbulence (the rate of turbulent kinetic en-
13 ergy dissipation, ε) from moorings. However, we show that for deployments
14 in the upper part of the water column, estimates of ε will be biased by the
15 vertical gradient in wave orbital velocities. To remove this bias, we develop a
16 modified structure function methodology, which exploits the differing length-
17 scale dependencies of the contributions to the structure function due to turbu-
18 lent and wave orbital motions. The success of the modified method is demon-
19 strated through comparison of ε estimates based on data from instruments at
20 three depths over a three month period under a wide range of conditions, with
21 appropriate scalings for wind stress and convective forcing.

22 1. Introduction

23 Exchanges of heat, freshwater and trace gases between the ocean and the atmosphere are critical
24 in regulating the climate and depend directly on the properties of the ocean surface boundary layer
25 (OSBL) (e.g. D’Asaro 2014; Franks 2014; Large et al. 1994). The structure of the OSBL depends
26 on turbulent processes that cannot be directly simulated in geographical scale numerical models
27 and which therefore have to be parameterized (Burchard et al. 2008; Belcher et al. 2012; Calvert
28 and Siddorn 2013).

29 Turbulence in the OSBL is widely recognised as being produced by wind-driven surface shear
30 stress, destabilising surface buoyancy fluxes and (in shelf seas) tidal current shear at the bottom
31 boundary (e.g. Brainerd and Gregg 1993; Simpson 1981). Other surface-driven processes include
32 breaking waves (e.g. Agrawal et al. 1992; Terray et al. 1996), Langmuir circulation (e.g. Thorpe
33 2004), submesoscale eddies (e.g. Taylor 2016) and swell waves (e.g. Wu et al. 2015). Developing
34 effective parameterizations for such diverse processes requires robust measurements under a wide
35 range of environmental conditions, presenting significant observational challenges.

36 The structure function method is an established technique for calculating the turbulent kinetic
37 energy (TKE) dissipation rate, ε , from velocity profiles such as those obtained with an acous-
38 tic Doppler current profiler (ADCP) (e.g. Wiles et al. 2006; Mohrholz et al. 2008; Lucas et al.
39 2014; Simpson et al. 2015; McMillan and Hay 2017). The method relates ε to the variance of
40 the along-beam turbulent velocity difference evaluated over a range of separation distances. In-
41 strument choice and configuration impose constraints on the data collected, but once configured,
42 ADCP can be deployed to make unattended long-term observations, unlike standard microstruc-
43 ture techniques.

44 Surface waves induce orbital motions within the water column, the speed of which reduce with
45 depth. The velocity associated with the orbital motions may be observed by the ADCP, potentially
46 affecting the structure function and introducing bias in the ϵ estimates. To date, the structure
47 function technique has typically been applied to observations from sites with small amplitude
48 surface waves or at depths unlikely to be affected by significant wave orbital velocities (Wiles
49 et al. 2006; Lucas et al. 2014; Simpson et al. 2015; McMillan and Hay 2017).

50 An exception is the application of the technique by Thomson (2012) to obtain ϵ estimates within
51 the crests of breaking waves by mounting the ADCP on a surface following Lagrangian float
52 and by necessity limiting the range of separation distances over which the structure function was
53 evaluated. Similarly, in order to measure vertical profiles of ϵ in the near-surface under breaking
54 waves, Sutherland and Melville (2015) adapted the technique by restricting both the range of
55 separation distances and the time-averaging period over which the statistical properties of the
56 structure function were evaluated. Restricting the range of separation distances minimises the
57 difference in the orbital velocity seen by different ADCP bins, whilst adopting a time averaging
58 period similar to or less than that of the waves will result in the wave orbital velocity being treated
59 as a background mean flow.

60 Working in a shallow water, wave-dominated environment, Whipple and Luetlich (2009) as-
61 sume that the velocity variance at each depth (calculated over a sampling period much longer than
62 the wave period) is dominated by the wave orbital velocity at that depth. They fit a theoretical
63 vertical profile based on linear wave theory to the observations in order to characterise the effec-
64 tive wave contribution to the structure function over a specified depth range. This is then used
65 to remove the influence of waves and isolate the much smaller turbulent signal. Whilst this ap-
66 proach explicitly recognises the contribution of the vertical gradient of the wave orbital velocity

67 to the structure function, it is only applicable in situations where the wave influence dominates the
68 structure function and does not lend itself to more general application.

69 The aims of this paper are, firstly, to demonstrate that ϵ estimates made using the standard struc-
70 ture function method with ADCP data are inherently susceptible to bias in the presence of surface
71 waves due to a contribution to the structure function from the vertical gradient in the speed of the
72 associated wave orbital motion; and secondly, to present a modification to the standard method
73 that addresses such bias. Section two briefly covers the underlying theory; demonstrates the stan-
74 dard method's bias using the wave orbital motions under synthetic monochromatic waves; and
75 describes the proposed modified method based on the application of linear wave equations. Sec-
76 tion three describes a set of long-term field observations from a shelf sea site that were used to test
77 the standard and modified methods. Section four uses established similarity scaling approaches to
78 compare the results under differing surface forcing conditions and section five is a discussion of
79 the results.

80 **2. Theory**

81 *a. Structure Function*

82 The theoretical basis of the structure function technique and its derivation from the Kolmogorov
83 similarity hypotheses is described in detail elsewhere (Sreenivasan 1991; Frisch 1995; Antonia
84 et al. 1997; Pope 2000; Lucas et al. 2014; McMillan and Hay 2017). In summary, the tech-
85 nique assumes that for isotropic turbulence in high Reynolds number flows, an inertial sub-
86 range of length scales exists over which there is a conservative cascade of energy from larger
87 to smaller motions. The statistical properties of the longitudinal turbulent velocity fluctuation,

88 $\delta u'(x, r) \equiv u'(x + r) - u'(x)$, where $u'(x)$ is the along-axis turbulent velocity at location x , then
 89 vary as a function of the separation distance r .

90 Invoking Taylor's frozen field hypothesis to allow sampling of the statistical properties of the
 91 flow over time, the mean $\delta u'$ is related to ε for r values within the inertial sub-range as:

$$\langle (\delta u')^n \rangle \propto \langle \varepsilon \rangle^{n/3} r^{n/3} \quad (1)$$

92 where the angle brackets indicate time averaging over a statistically valid sampling period and n
 93 is the order of the structure function (Kolmogorov 1991a,b; Pope 2000).

94 The second-order structure function, $D_{LL}(x, r)$, is then defined as:

$$D_{LL}(x, r) \equiv \langle [u'(x + r) - u'(x)]^2 \rangle \quad (2)$$

95 and for values of r within the inertial sub-range $D_{LL}(x, r)$ is related to $\varepsilon(x)$ as:

$$D_{LL}(x, r) \propto C_2 \varepsilon^{2/3} r^{2/3} \quad (3)$$

96 where C_2 is a universal constant of proportionality, frequently taken to be 2.1 based on atmospheric
 97 studies (Wiles et al. 2006; Lucas et al. 2014; Simpson et al. 2015), whilst McMillan and Hay (2017)
 98 use 2.0 based on both theoretical considerations and the comparison of ε estimates made using the
 99 structure function and spectral integral methods.

100 From (3), the second-order structure function exhibits a length-scale dependence on $r^{2/3}$, so a
 101 least-squares linear regression of $D_{LL}(x, r)$ against $r^{2/3}$, at fixed x , gives:

$$D_{LL}(x, r) = A_0 + A_1 r^{2/3} \quad (4)$$

102 where A_0 is a measure of the Doppler and instrument noise and A_1 is the gradient of the linear
 103 regression over the range of r evaluated. From (3), $A_1 = C_2 \varepsilon^{2/3}$, which then gives an estimate of ε
 104 at x for the sampling period as:

$$\varepsilon = \left(\frac{A_1}{C_2} \right)^{3/2} \quad (5)$$

105 When applied to ADCP data, a sampling period of several minutes is typically used, during which
106 multiple individual velocity profiles are collected at a frequency of 1 – 2 Hz. The along-beam
107 velocity data is processed for each beam separately, with the along-beam turbulent velocity, u' ,
108 calculated for each bin by deducting its mean over the sampling period in order to remove the
109 mean flow and hence any background shear.

110 The structure function, D_{LL} , is then calculated from the velocity differences at separation dis-
111 tances, r , based on multiples of the along-beam bin size. The minimum separation is taken as
112 two bins due to the lack of independence in the velocities measured in adjacent bins (Teledyne RD
113 Instruments 2014). The squares of the velocity differences are then averaged over the sampling pe-
114 riod as in (2). Using a central difference scheme (e.g. Wiles et al. 2006), D_{LL} is evaluated for each
115 bin for separation distances centred on the bin, with the r values that can be resolved dependent
116 on the bin's position within the range of bins for which the turbulent velocity is available.

117 A maximum separation distance, r_{\max} , is specified for the regression of D_{LL} against $r^{2/3}$. This
118 should be chosen to include as much of the inertial sub-range as possible, although in practice
119 the configuration of the ADCP may restrict the range over which turbulent velocities are resolved.
120 When this isn't a constraint, r_{\max} must not exceed the upper length limit of the inertial sub-range,
121 beyond which D_{LL} is expected to tend towards a constant. The selection of r_{\max} therefore depends
122 on both instrument constraints and the turbulent properties of the observed flow.

123 *b. Wave Orbital Motion*

124 A basic representation of deep-water surface gravity waves is to treat them as sinusoidal, with
125 amplitude A , wavelength λ and period T , giving a radian frequency of $\omega = 2\pi/T$, wavenumber
126 $k = 2\pi/\lambda$ and phase speed c given by $c^2 = \omega^2/k^2 = g/k$, with g being the acceleration due to
127 gravity.

128 The simplest model for the motion in the water column below such waves (e.g. Phillips 1977;
 129 Simpson and Sharples 2012), is of non-rotational circular motion with a speed at depth z (zero at
 130 surface, positive up) of:

$$v_{\max} = \omega A e^{kz} \quad (6)$$

131 Over a vertical distance δz around depth z_0 , the difference in the speed of the orbital motion is:

$$\begin{aligned} \delta v_{\max}(z_0) &= \omega A \left[e^{k(z_0 + \delta z/2)} - e^{k(z_0 - \delta z/2)} \right] \\ &\approx k v_{\max}(z_0) \delta z \end{aligned} \quad (7)$$

132 subject only to the adoption of the small angle approximation that $\sinh(k\delta z/2) \approx k\delta z/2$, which is
 133 valid to within 2% for $\delta z < \lambda/10$. Hence, at all depths, the vertical difference in the orbital speed
 134 varies linearly with the vertical separation distance.

135 As illustrated in figure 1, this vertical variation in the speed of the orbital motion will result in a
 136 contribution to the structure function even in the absence of turbulence. Under a monochromatic
 137 wave, the along-beam velocity measured in the ADCP bins will vary sinusoidally in phase in all
 138 bins, but with an amplitude that depends on the depth of the bin. Since the sampling period used
 139 to determine the structure function is normally much longer than the surface wave period (several
 140 minutes versus typically less than 15 seconds), the mean of the along-beam component of the
 141 wave orbital motion measured by any bin is \sim zero and will not contribute to the mean velocity
 142 deducted to calculate the fluctuating turbulent along-beam velocity u' . Consequently, u' retains the
 143 along-beam component of the time-varying wave orbital motion. Any differences in u' between
 144 bins will be treated as a turbulent velocity variation when calculating D_{LL} , potentially resulting in
 145 a bias in the calculated ε estimates.

146 In order to quantify the potential bias, ε values were calculated using wave orbital velocities cal-
 147 culated from linear wave theory for a range of monochromatic waves with amplitudes and periods

representative of an exposed shelf-sea environment. These synthetic wave orbital velocities were calculated for the bin locations of virtual ADCP at depths of 20, 35 and 50 m with an upward-looking orientation, sampling via a beam with a 20° beam angle (inclination from the vertical) with 30 bins at a 0.1 m vertical bin spacing and bin one centred at 0.97 m from the transducer. The measurement frequency was 1 Hz with a sampling period of 300 s resulting in 300 velocity profiles.

Assuming waves propagating in the x direction and the ADCP beam in the $y = 0$ plane, the horizontal (u) and vertical (w) velocities vary as:

$$\begin{aligned} u &= \omega A e^{kz} \sin(kx - \omega t) \\ w &= -\omega A e^{kz} \cos(kx - \omega t) \end{aligned} \quad (8)$$

with t being time.

The along-beam velocities in each bin were calculated by applying a rotation matrix based on the virtual ADCP beam geometry (Teledyne RD Instruments 2010). The structure function, D_{LL} , was calculated using a central difference scheme and ε estimates were determined for each bin from the regression of D_{LL} against $r^{2/3}$ with r_{\max} equal to 2.0 m. Beam average ε values were calculated as the geometric mean of the individual values for all bins for which the structure function was resolved for all $r \leq r_{\max}$.

Figure 2 shows the beam average ε estimates for each of the three instruments for surface waves with amplitudes up to 2 m and periods between 7 and 13 s. The bias in ε is more than $1 \times 10^{-5} \text{ W kg}^{-1}$ for an ADCP at a depth of 20 m under waves with an amplitude of 1.8 m and a period of 8 s. Even for an instrument at 50 m depth, swell waves with a period of 11-12 s and an amplitude of 1.6 m could potentially introduce a bias of $\mathcal{O} 10^{-7} \text{ W kg}^{-1}$, two orders of magnitude above the expected noise floor (Lucas et al. 2014).

169 The bias in ε depends on the difference in the speed of the wave orbital motion over distance
170 r_{\max} , which depends on both the amplitude and the attenuation rate of the speed of the orbital
171 motion. Since the attenuation rate depends on wave number, the period of the waves contributing
172 most to any bias will typically increase with ADCP depth.

173 For a spectrum of waves, linear wave theory would suggest that the along-beam velocities ob-
174 served by the ADCP will be the sum of the wave orbital velocities due to the various component
175 waves. Whilst the velocity contribution from each component wave will depend on its surface
176 properties and attenuation rate, each will exhibit the linear variation with vertical separation in
177 (7). The composite wave orbital velocity can therefore also be expected to demonstrate a linear
178 length-scale dependency.

179 Though the leading order water motions associated with the surface waves are periodic and do
180 not affect the time-averaged current profile. Surface waves also produce a second order, depth-
181 varying Lagrangian transport in their direction of propagation, the Stokes drift (e.g. Phillips 1977;
182 Ardhuin et al. 2009). Within the structure function calculation, any non-periodic velocity observed
183 by an ADCP bin is considered as part of the mean flow and removed when the turbulent velocity
184 is calculated. Asymmetric periodic flows, such as the difference between the upper and lower
185 portions of a wave orbital motion that leads to Stokes drift, may result in a non-zero contribution
186 to the mean flow as well as a contribution to the structure function based on the depth dependent
187 variation in the periodic motion. The Stokes drift speed decays exponentially with depth at twice
188 the rate of the wave orbital motion (Phillips 1977). It is therefore also expected to exhibit a linear
189 length-scale dependence over a limited vertical separation distance.

190 Exploiting the differing length-scale dependencies of the turbulent and wave-related components
191 of the observed velocity offers the possibility of separating these two components of the structure
192 function.

193 *c. Modified Methodology to Reject Impact of Wave Orbital Motion*

194 From (1), the n^{th} order structure function varies as $r^{n/3}$, hence D_{LL} will vary linearly against
 195 $r^{2/3}$. By contrast, from (7), the difference in the maximum wave orbital velocity magnitude δv_{max}
 196 varies linearly with r , hence from (2), the contribution to D_{LL} varies as r^2 . In the regression of D_{LL}
 197 against $r^{2/3}$, the contribution to the structure function from the vertical variation in wave orbital
 198 velocity will therefore increase as $(r^{2/3})^3$.

199 The differing rates at which the contribution of the turbulent and wave orbital motion compo-
 200 nents of the structure function vary with separation distance provides the basis for the modified
 201 method. Instead of the standard least-squares linear regression of D_{LL} against $r^{2/3}$ as in (4), a
 202 least-squares fit is done to determine the coefficients for the linear model:

$$D_{LL}(x) = A_0 + A_1 r^{2/3} + A_3 (r^{2/3})^3 \quad (9)$$

203 The modified method essentially assumes that the wave orbital motion and turbulence do not in-
 204 teract and the associated velocities are simply additive. The contribution to D_{LL} due to the vertical
 205 gradient in the speed of the wave orbital motion (contained in the A_3 coefficient) can therefore be
 206 extracted without affecting the turbulent contribution. Hence the A_0 coefficient continues to de-
 207 scribe the instrument and Doppler noise and the A_1 coefficient continues to describe the turbulence,
 208 with ε still calculated using (5).

209 The effectiveness of the modified method was tested by applying it to the synthesized wave
 210 orbital velocity data described in section 2b. Figure 3 shows the regression of D_{LL} against $r^{2/3}$
 211 for both the standard and modified methods for the instrument at depth 35 m with a surface
 212 wave of amplitude 1 m and a period of 10 s. The standard method results in a calculated ε of
 213 $1.4 \times 10^{-7} \text{ W kg}^{-1}$ and a physically meaningless negative A_0 value of $-2.6 \times 10^{-5} \text{ m}^2 \text{ s}^{-2}$. By

contrast, the A_0 and A_1 coefficients for the modified method correctly reflect the fact that there was no turbulent motion or system noise in the synthesized velocity data.

d. Similarity Scaling

In order to compare the results of the standard and modified methods at different depths and under widely varying environmental conditions, two distinct surface forced regimes with established similarity scalings are considered. The relevant scaling factors are applied to ε estimates calculated using both the standard and modified methods to illustrate the conformance of the results from the two methods to the standard scalings.

1 Wind stress forcing. Following Monin-Obukhov similarity theory, a local balance is assumed between ε and TKE production based on a constant stress “law of the wall” relationship (Anis and Moum 1995; Lombardo and Gregg 1989; Brainerd and Gregg 1993; Lozovatsky et al. 2005; Tedford et al. 2014; Bogucki et al. 2015; D’Asaro 2014). This results in a scaling factor ε_s given by:

$$\varepsilon_s = -\frac{u_*^3}{\kappa z} \quad (10)$$

where u_* is the friction velocity in the water, calculated as $u_* = (\tau_s/\rho_0)^{1/2}$ for surface wind stress τ_s and water density ρ_0 ; κ is the von Kármán constant (0.41); and z is depth (zero at surface, positive up). Within the mixed layer, but below the region of direct impact from breaking waves (Agrawal et al. 1992; Anis and Moum 1995), ε estimates would be expected to scale as $\varepsilon/\varepsilon_s \approx 1$, with reported values typically in the range 1 - 2 based on limited duration observations (Lombardo and Gregg 1989; Lozovatsky et al. 2005; Shay and Gregg 1986; Thorpe 2005).

234 **2 Convective forcing.** By convention a positive surface buoyancy flux, $B_0 > 0$, indicates a loss
235 of heat from the ocean surface to the atmosphere, increasing the ocean surface density and cre-
236 ating unstable conditions leading to convection and an increase in ε . Within the mixed layer,
237 but below the Monin-Obukhov length (the depth at which wind stress forcing and convective
238 forcing match), ε is expected to be constant, reducing only at the base of the mixed layer
239 when it encounters stratification and contributes to mixing by entrainment (Shay and Gregg
240 1986; Lombardo and Gregg 1989). Hence under low wind conditions, ε estimates would be
241 expected to scale as $\varepsilon/B_0 \approx 1$, with reported values based on limited duration observations
242 typically being in the range 0.5 to 0.8 under conditions of both sustained and diurnal con-
243 vection, with some indication of a time dependence as convection becomes established (Anis
244 and Moum 1992; Brainerd and Gregg 1993; Lombardo and Gregg 1989; Shay and Gregg
245 1984a,b, 1986; Thorpe 2005).

246 Combined scalings incorporating both wind stress and convective forcing have been developed
247 as linear combinations of the scalings for the individual forcing regimes (e.g. Lombardo and Gregg
248 1989; Tedford et al. 2014). However, the variation in the reported weighting coefficients suggests
249 that the combined scaling may be less robust than the scaling for the individual regimes. The
250 objective of the current study is not to revisit these scalings, but to use them as the basis for com-
251 paring the susceptibility of the standard and modified structure function methods to wave-induced
252 bias. The scalings were therefore applied separately to ε estimates based on field observations
253 made under the relevant forcing conditions and the results compared to a default depth-constant
254 unity reference value.

3. Observations

a. Dataset

The present analysis is based on observations made during the period January to March 2015 from a site in the Celtic Sea. The site has a water depth of ~ 150 m; is more than 200 km from any coast, removing it from the direct coastal influences; and is over 125 km from the shelf edge, minimising the impact of any shelf break processes. The wave climate included both locally generated waves and remotely generated swell, unaffected by significant shoaling or coastal reflections.

Three Teledyne RD Instruments 600 kHz Workhorse ADCP were deployed on a buoyancy tensioned mooring attached to a seabed anchor weight. The instruments were all configured in pulse-to-pulse coherent mode (mode 5) (Teledyne RD Instruments 2014) with a sampling frequency of 1 Hz and one ping per ensemble (no ensemble averaging), with a vertical bin size of 0.1 m and bin one centred 0.97 m from the transducer. The instruments operated for a five-minute sampling period, followed by a 15-minute rest interval, resulting in three sampling periods per hour, each comprising 300 velocity profiles for each of the four beams. The uppermost instrument had a 20° beam angle and was deployed upward-looking; the middle instrument also had a beam angle of 20° , but was deployed downward-looking; whilst the lowest instrument had a beam angle of 30° and was upward-looking.

The mooring rotated with the tide, the depth-averaged current having spring tide maxima of $\sim 0.5 \text{ m s}^{-1}$ with a pronounced spring-neap cycle. The instruments' measurement volumes were centred at mean depths of ~ 24.0 , 42.5 and 52.5 m. Reliable velocity measurements were typically returned for bins 1 to 30 for the 20° beam angle instruments and bins 1 to 28 for the 30° beam angle instrument, equating to bin centres at along-beam distances of ~ 1 to ~ 4.2 m from the transducer.

278 Three additional moorings provided supplementary information used in this analysis. All moor-
279 ings were located within 1 km of each other throughout the observation period. One of the moor-
280 ings provided full water column temperature, salinity and density (Wihsgott et al. 2016). Another
281 was a UK Met Office ODAS buoy, which provided meteorological and wave data including hourly
282 measurements of average wind speeds and direction plus maximum gust speeds at 3 m above
283 the sea surface based on sampling over a 10 minute period; air and sea surface temperature; at-
284 mospheric pressure and relative humidity; plus significant wave height and average wave period
285 based on 17.5 minutes of observations. The third was a UK Centre for Environment, Fisheries and
286 Aquaculture Science (Cefas) SmartBuoy, which provided half hourly sea surface temperature and
287 salinity, plus photosynthetically active radiation (used as a proxy for solar irradiance).

288 *b. Data Analysis*

289 Surface stress and buoyancy flux were calculated using the TOGA COARE 3 bulk flux algo-
290 rithm, taking account of the heights of the instruments on the ODAS buoy (Fairall et al. 2003).

291 The ADCP beam coordinate turbulent velocities, u' , were calculated independently for each bin
292 in each beam by deducting the mean for that bin over the sampling period. Outlier values were
293 identified by comparison with the rms value of all turbulent velocities for all bins and beams in
294 the current sampling period and rejected. Outliers were almost exclusively in the furthest bin for
295 which the velocity was resolved.

296 The second-order structure function, D_{LL} , was calculated using a central difference scheme over
297 all resolvable separation distances, $r = r_j \Delta r$, where r_j is the separation in number of bins and Δr is
298 the along-beam bin size determined by the vertical bin size and the beam angle. For even number

bin separations, $r_j = 2, 4, 6 \dots$ around bin i :

$$D_{LL}(x_i, r_j \Delta r) = \left\langle \left[u' \left(x_i + \frac{r_j}{2} \Delta r \right) - u' \left(x_i - \frac{r_j}{2} \Delta r \right) \right]^2 \right\rangle \quad (11)$$

where $u'(x_i)$ is the turbulent velocity in the bin centred at distance x_i from the transducer. For odd number bin separations, $r_j = 3, 5, 7 \dots$, the average of the two possible combinations was used, so that:

$$D_{LL}(x_i, r_j \Delta r) = \left\langle \frac{1}{2} \times \left[u' \left(x_i + \text{floor} \left(\frac{r_j}{2} \right) \Delta r \right) - u' \left(x_i - \text{ceil} \left(\frac{r_j}{2} \right) \Delta r \right) \right]^2 + \frac{1}{2} \times \left[u' \left(x_i + \text{ceil} \left(\frac{r_j}{2} \right) \Delta r \right) - u' \left(x_i - \text{floor} \left(\frac{r_j}{2} \right) \Delta r \right) \right]^2 \right\rangle \quad (12)$$

where floor() (ceil()) means round down (up) to the integer.

The D_{LL} values for all bins were used in least-squares fit regressions against $r^{2/3}$, to give a beam aggregate ε value for the sampling period for both the standard (4) and modified (9) methods. The regressions were repeated for a range of r_{\max} values between 0.8 and 3.0 m (the maximum possible given the instrument configurations). Basic result screening rejected regressions if the coefficients did not produce a strictly increasing result for $r > 0$. Equation (5) was used to calculate ε with C_2 as 2.0. The geometric mean of the individual beam values provided a single representative ε data point per sampling period for each instrument, method and r_{\max} value over the three months of observations, resulting in approximately 6,500 data points for each combination of instrument, method and r_{\max} .

The adjusted coefficient of determination, $R_{\text{adj}}^2 = 1 - (1 - R^2) \left[\frac{m-1}{m-(p+1)} \right]$, where R^2 is the unadjusted coefficient of determination; m is the sample size; and p is the number of independent variables in the regression, was calculated for each regression. Using R_{adj}^2 rather than R^2 allows the quality of the fit from both the standard and modified methods to be compared directly, taking account of the additional term in the modified method.

4. Results

The three months of observations included in this analysis cover a wide range of winter conditions. Throughout the period, the water column was negligibly stratified. The surface buoyancy flux, B_0 , was characterised by a destabilising heat flux to the atmosphere (B_0 positive) approximately 70% of the time, when the mean flux was $6 \times 10^{-8} \text{ W kg}^{-1}$ and the maximum $1.9 \times 10^{-7} \text{ W kg}^{-1}$. Solar irradiance resulted in intermittent diurnal stabilising (B_0 negative) buoyancy fluxes, centred around midday and increasing in duration and maximum intensity over the period of the observations. It is anticipated that this warming may have resulted in short periods of diurnal surface stratification under low wind stress conditions, therefore observations under these conditions were excluded from the analysis.

Wind speeds (at 3 m) had a range from 1 to 19 m s^{-1} with a rms of 9.2 m s^{-1} and maximum gusts of 28 m s^{-1} . Significant wave height varied between 1.2 and 14.1 m with a rms value of 5.3 m, whilst the average wave period varied between 4.4 and 14.4 s, with a rms of 8.0 s. The resulting surface wind stress, τ_s , varied between 2×10^{-4} and 1.2 Pa, with a rms of 0.27 Pa.

The ϵ estimates were sorted according to the forcing conditions at the time of the observation, without any reference to adjustment time scales, resulting in the following datasets:

- Wind stress forcing: $\tau_s > 0.05 \text{ Pa}$ giving $\sim 5,300$ data points per instrument for each model and r_{max} evaluated (81.9% of observations)
- Convective forcing: $\tau_s \leq 0.05 \text{ Pa}$ and $B_0 > 0$ giving ~ 870 data points per instrument for each model and r_{max} evaluated (13.4% of observations)

The number of observations varied slightly between instruments and between methods, with the modified method having the same or fewer ϵ estimates for each instrument. Observations made under conditions when $\tau_s \leq 0.05 \text{ Pa}$ and $B_0 \leq 0$ (i.e. low wind and surface heating) comprised

341 4.7% of observations and were excluded from the current analysis. The τ_s threshold was chosen
342 based on the overall distributions of τ_s and B_0 , without any structured attempt at optimisation.

343 *a. Observation of Wave Orbital Motion*

344 Periodic variations were clearly apparent in much of the along-beam velocity data from each of
345 the ADCP and were coherent across all bins in a beam. Fourier analysis typically showed a peak at
346 or around the average surface wave period. In order to test whether the observations demonstrated
347 the vertical gradient expected of wave orbital motion, the ADCP data was transformed from beam
348 to earth coordinates and the rms of the earth coordinate vertical velocity, w_{rms} , and the difference,
349 δw_{rms} , over a vertical separation distance, δz , of 2.0 m, was calculated for each instrument and
350 for each five-minute sampling period. The theoretical variation in the wave orbital speed, δv_{max} ,
351 was calculated over δz at each instrument's observation depth using (7), assuming monochromatic
352 waves of amplitude equal to half of the concurrent significant wave height and with the observed
353 average period.

354 Figure 4 plots δw_{rms} versus δv_{max} together with the linear regression for each instrument. De-
355 spite the simplistic assumption of monochromatic waves in the calculation of δv_{max} , all three
356 instruments demonstrate a linear relationship with nearly identical coefficients over the full range
357 of conditions. The robust correlation between δw_{rms} and δv_{max} , which are derived from indepen-
358 dent datasets, indicates that wave orbital motions are producing a vertical gradient in the velocity
359 profiles measured by the ADCP in a manner consistent with the simple theoretical model assumed.

360 *b. Comparison of the Standard and Modified Methods*

361 Figure 5 summarises the results for the standard and modified methods for all three instruments
362 and under both surface wind stress and convective forcing. All regressions are based on $r_{\text{max}} \sim$

363 2.0 m, the exact value depending on the separation distances evaluated given the ADCP geometry.
364 The results for the two forcing processes are considered separately:

365 **1. Wind stress forcing.** The median wind stress scaled ε estimates for each instrument and
366 for both the standard and modified methods are shown in panel (a) of figure 5 and the data is
367 summarised in table 1. For the standard method, the median scaled ε estimates vary from 9.15
368 for the uppermost instrument to 1.78 for the lowest instrument, with a clear depth dependence.
369 Over 45% of standard method ε estimates at 24 m have a bias of an order of magnitude or
370 greater compared with the default unity scaling, with $> 97\%$ of observations exhibiting a
371 bias of two or more. The bias decreases with depth, although over 45% of the observations
372 at 52.5 m remain subject to a bias of two or more. In contrast, for the modified method,
373 the median scaled ε estimates vary between 1.11 and 0.69 for the three instruments, with no
374 apparent depth dependence, suggesting no significant departure from the “law of the wall”
375 unity scaling.

376 **2. Convective forcing.** The median surface buoyancy flux scaled ε estimates for each instru-
377 ment and for both the standard and modified methods are shown in panel (b) of figure 5 and
378 the data is summarised in table 2. The standard method median bias is higher for all instru-
379 ments than the equivalent bias for the surface shear stress scaled observations, varying from
380 21.15 for the uppermost instrument to 2.21 for the lowest instrument and again demonstrat-
381 ing a clear depth dependence. In contrast, for the modified method, the median scaled ε
382 estimates vary between 1.36 and 0.79 for the three instruments and again exhibit no apparent
383 depth dependency, suggesting no significant departure from the unity scaling with B_0 .

384 *c. Method Sensitivity to Selection of r_{\max}*

385 In principle, it is desirable to evaluate the structure function regression over as much of the
386 inertial sub-range as possible in order to better determine ε , subject to the constraint on r_{\max} being
387 less than the upper limit of the inertial sub-range.

388 The sensitivity of the standard and modified methods to the choice of r_{\max} is illustrated in figure 6
389 for both wind stress and convective forcing with r_{\max} as close as possible to 1, 2 and 3 m. All of
390 these r_{\max} values are expected to be within the inertial sub-range given the water column density
391 structure and turbulence levels. For $r_{\max} \sim 1$ m, the regression of D_{LL} against $r^{2/3}$ uses data for
392 just eight separation distances (from two bins to nine bins). The number of separation distances
393 increases approximately linearly with r_{\max} , subject to the dependence of the along-beam bin centre
394 spacing on beam angle. For $r_{\max} \sim 2$ m (3 m), the regression uses data for 18/16 (27/25) separation
395 distances for the $20^\circ/30^\circ$ instrument beam angles.

396 For the standard method, reducing r_{\max} reduces the bias but does not eliminate it. Even with
397 r_{\max} reduced to 1 m, the median bias for observations at 24 m remains 4.2 for wind stress forcing
398 and 8.2 for convective forcing. However, reducing r_{\max} to 1 m does reduce the median bias to less
399 than two for the observations at 42.5 m and 52.5 m for both forcing regimes.

400 The impact of reducing r_{\max} on the quality of the fit for the regression of D_{LL} against $r^{2/3}$ and
401 therefore on the confidence in the calculated ε estimate is shown in table 3 for wind stress forcing
402 and table 4 for convective forcing. Reducing r_{\max} from ~ 2 m to ~ 1 m dramatically reduces the
403 mean R_{adj}^2 values.

404 For the modified method, varying r_{\max} has only minimal impact on the median scaled ε estimates
405 for all three depths and both forcing regimes. The difference in the median scaled ε values is
406 negligible for $r_{\max} \sim 1$ m and 2 m, with the values for $r_{\max} \sim 3$ m being fractionally lower. The

407 R_{adj}^2 values for the modified method consistently indicate a better fit than the standard method,
408 although the difference is negligible for $r_{\text{max}} \sim 1$ m, only becoming significant with increasing
409 r_{max} .

410 *d. Wave Information from the Modified Method*

411 The additional regression coefficient produced by the modified method (A_3) is expected to be
412 dependent on the vertical difference in the speed of the wave orbital motion over the distance r_{max}
413 at the observation depth of the ADCP. Figure 7 plots the A_3 coefficient for each regression for
414 each instrument against the square of the difference in the theoretical wave orbital speed based
415 on the concurrent surface wave observations (δv_{max}), as described in section 4a, as well as linear
416 regressions for each instrument.

417 The scatter in figure 7 is considered to result from the assumption of monochromatic waves, with
418 the average period of the surface waves not being fully representative of the spectrum of waves
419 contributing to the vertical gradient in the wave orbital speed at the ADCP depths. However,
420 despite this simplification, the clear linear relationship between the A_3 coefficient and $(\delta v_{\text{max}})^2$.
421 suggests that the modified method is extracting the contribution to the structure function due to the
422 vertical variation in the wave orbital velocity speed as expected.

423 A specific δv_{max} cannot be attributed to a unique surface wave condition, even under the as-
424 sumption of monochromatic waves, since waves with different amplitudes and wavelengths could
425 produce the same vertical velocity difference. In principle it may be possible to use the variation
426 of A_3 with depth to determine an "effective" surface monochromatic wave, but this is beyond the
427 scope for the current study.

5. Discussion

Whilst three decades of ocean turbulence measurements using ship based microstructure profilers have provided strong quantitative links between the dissipation of turbulence kinetic energy and its forcing, the full geographic and temporal variability of turbulence, and hence mixing, remains a first order problem in oceanographic research (Ivey et al. 2008; Moum and Rippeth 2009; Mead Silvester et al. 2014). Part of the solution to this problem has been the development of new techniques for measuring longer time series of turbulence parameters. Amongst the more successful has been the application of moored off-the-shelf acoustic Doppler current profilers (ADCP), initially through the development of the variance method (Stacey et al. 1999; Lu and Lueck 1999; Rippeth et al. 2002), but more recently through a structure function approach (Wiles et al. 2006; Lucas et al. 2014).

In particular the structure function technique is an attractive option as the turbulence estimates are not sensitive to instrument motion, and can therefore be made mid-water column from moored platforms (Lucas et al. 2014), avoiding the specific processing to remove platform motion required for spectral techniques (Bluteau et al. 2016). Furthermore the development of pulse-to-pulse coherent operating modes has enabled reliable estimates of ϵ down to a noise floor estimated as $\sim 3 \times 10^{-10} \text{ W kg}^{-1}$ (Lucas et al. 2014). However, the averaging period implicit in the structure function technique is long relative to the period of surface waves, potentially leading to a bias in ϵ estimates due to the variation of the speed of the wave orbital motion with depth.

Here we have demonstrated the degree to which ϵ is biased by the presence of surface waves using synthetic wave data. We have then developed a modified second-order structure function method which exploits the differing length-scale dependencies of the contributions due to turbulent and wave orbital motions in order to remove the surface wave influence. The standard and modified

451 methods were then tested using data collected over a three-month winter period by three ADCP
452 operating in pulse-to-pulse coherent mode and mounted on a mooring at different depths. The
453 observational period provided a wide range of wind, wave and surface buoyancy flux conditions.

454 Estimates of ε made using both the standard and modified structure function methods were then
455 scaled using established scaling for either wind stress or convective forcing. The results using
456 the standard method show a significant departure from the expected value under both forcing
457 conditions. The bias is greatest for the uppermost instrument and declines significantly with depth.
458 This accords with the hypothesis that the bias results from the vertical gradient in the speed of the
459 wave orbital motions, which decay exponentially with depth. The median bias for convective
460 forcing scaled ε estimates were higher than those scaled for wind stress forcing at all depths,
461 indicating that the bias due to surface waves is more significant under relatively lower turbulence
462 conditions. In contrast, the scaled ε estimates obtained using the modified method collapse to
463 \sim unity for the observations under both wind stress and convective forcing, indicating that the ε
464 profiles are in approximate accordance with the nominal scaling.

465 Analysis of the length-scale dependence of the speed of wave orbital motions for intermediate
466 depth waves (see Appendix) suggests that the modified method should also be effective in remov-
467 ing bias in ε estimates from observations affected by surface waves in shallower water, providing
468 the orbital motions match standard wave theory. However, pending evaluation against actual ob-
469 servations, care is needed in applying the modified method in shallow water conditions.

470 These results lead to the conclusions that:

- 471 • There is significant potential for bias in second-order structure function estimates of ε as a
472 result of the depth variation of surface wave orbital velocities.

- A modified method, which exploits the differing length-scale dependencies of the contributions to the structure function from turbulent and wave orbital motions, is effective in removing the surface wave bias in the ε estimates made under both wind stress and convective forced conditions.

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APPENDIX

Application with generalised wave equations

The generalised equations for the motion under surface waves describe elliptical orbits with an eccentricity that depends on the wave's wavelength, the water depth and the depth of the observation point. The horizontal and vertical velocity components under an infinitesimal monochromatic sinusoidal wave travelling in the x direction are given by:

$$\begin{aligned} u &= \frac{gk}{\omega} A \frac{\cosh(k(z+h))}{\cosh(kh)} \sin(kx - \omega t) \\ w &= -\frac{gk}{\omega} A \frac{\sinh(k(z+h))}{\cosh(kh)} \cos(kx - \omega t) \end{aligned} \quad (A1)$$

where g is acceleration due to gravity; k is wavenumber given by $k = 2\pi/\lambda$ and λ is wavelength; ω is radian frequency given by $\omega = 2\pi/T$ and T is wave period; z is depth, with $z = 0$ at the sea surface and positive upwards; h is water depth so that $z = -h$ at seabed; and t is time (Phillips 1977).

A vertically oriented ADCP with a beam in the $y = 0$ plane will see an along-beam velocity b_0 in the bin centred at $x = x_0$ and $z = z_0$ with contributions from both components depending on the beam angle θ , which is given by:

$$b_0 = \frac{gkA}{\omega \cosh(kh)} \left[\sin \theta \cosh(k(z_0 + h)) \sin(kx_0 - \omega t) - \cos \theta \sinh(k(z_0 + h)) \cos(kx_0 - \omega t) \right] \quad (A2)$$

The velocity difference δb_0 over a vertical range δz around depth z_0 will therefore be:

$$\delta b_0 = \frac{gkA}{\omega \cosh(kh)} \left[\begin{aligned} & \left[\sin \theta \cosh \left(k \left(z_0 + \frac{\delta z}{2} + h \right) \right) \sin(kx_{z_0 + \frac{\delta z}{2}} - \omega t) \right. \\ & \quad \left. - \cos \theta \sinh \left(k \left(z_0 + \frac{\delta z}{2} + h \right) \right) \cos(kx_{z_0 + \frac{\delta z}{2}} - \omega t) \right] \\ & - \left[\sin \theta \cosh \left(k \left(z_0 - \frac{\delta z}{2} + h \right) \right) \sin(kx_{z_0 - \frac{\delta z}{2}} - \omega t) \right. \\ & \quad \left. - \cos \theta \sinh \left(k \left(z_0 - \frac{\delta z}{2} + h \right) \right) \cos(kx_{z_0 - \frac{\delta z}{2}} - \omega t) \right] \end{aligned} \right] \quad (A3)$$

where $x_{z_0 - \frac{\delta z}{2}}$ is the x coordinate of the observation bin centred at $z = z_0 - \frac{\delta z}{2}$. For θ values of 20° or 30° and δz appropriate for r_{\max} values used with the structure function regression, the horizontal bin displacement $x_{z_0 + \frac{\delta z}{2}} - x_{z_0 - \frac{\delta z}{2}}$ will be $\ll \lambda$, so that $kx_{z_0 + \frac{\delta z}{2}} \approx kx_{z_0 - \frac{\delta z}{2}} \approx kx_0$ and the

503 orbital velocity observed in all bins is in phase. Equation (A3) then simplifies as:

$$\begin{aligned} \delta b_0 = \frac{gkA}{\omega \cosh(kh)} \Bigg[& \sin \theta \sin(kx_0 - \omega t) \Bigg[\cosh \left(k \left(z_0 + \frac{\delta z}{2} + h \right) \right) \\ & - \cosh \left(k \left(z_0 - \frac{\delta z}{2} + h \right) \right) \Bigg] \\ & - \cos \theta \cos(kx_0 - \omega t) \Bigg[\sinh \left(k \left(z_0 + \frac{\delta z}{2} + h \right) \right) \\ & - \sinh \left(k \left(z_0 - \frac{\delta z}{2} + h \right) \right) \Bigg] \Bigg] \end{aligned} \quad (\text{A4})$$

504 Applying the double angle hyperbolic identities and recognising that \cosh (\sinh) is an even (odd)
505 function, equation (A4) simplifies as:

$$\begin{aligned} \delta b_0 = \frac{gkA}{\omega \cosh(kh)} 2 \sinh \left(k \frac{\delta z}{2} \right) \Bigg[& \sin \theta \sin(kx_0 - \omega t) \sinh(k(z_0 + h)) \\ & - \cos \theta \cos(kx_0 - \omega t) \cosh(k(z_0 + h)) \Bigg] \end{aligned} \quad (\text{A5})$$

506 Grouping all the terms independent of δz into a function, F :

$$\begin{aligned} F = \frac{gkA}{\omega \cosh(kh)} \Bigg[& \sin \theta \sin(kx_0 - \omega t) \sinh(k(z_0 + h)) \\ & - \cos \theta \cos(kx_0 - \omega t) \cosh(k(z_0 + h)) \Bigg] \end{aligned} \quad (\text{A6})$$

507 equation (A5) becomes:

$$\delta b_0 = 2F \sinh \left(k \frac{\delta z}{2} \right) \quad (\text{A7})$$

508 For $k\delta z \ll 1$, the approximation $\sinh(x) \approx x$ can be applied, giving:

$$\delta b_0 \approx kF \delta z \quad (\text{A8})$$

509 For deep water waves, $\sinh(k(z_0 + h)) \approx \cosh(k(z_0 + h)) \approx \cosh(kh)$, so that equations (A6) and
510 (A2) become identical and (A7) becomes $\delta b_0 \approx kb_0 \delta z$, recovering equation (7).

511 More generally, equation(A8) suggests that whilst F may vary with z , δb_0 will vary linearly
512 with δz irrespective of the water depth, providing the wave orbital motion is described by the

513 generalised equations (A1), subject only to the constraint of δz being small relative to λ . This
514 suggests that the modified method has the potential to be effective at removing bias due to wave
515 orbital motion from ε estimates over a wider range of water depths.

516 *a. Testing the modified method for non-deep water waves*

517 It is reasonable to anticipate that there will be limits on the effectiveness of the modified method
518 as the water depth reduces. In order to test this, synthetic velocity data was generated for waves
519 with a range of wavelengths and amplitudes in different water depths, in the same manner as
520 described in section 2b, but using the general wave orbital motion equations (A1) rather than the
521 deep water equations (8).

522 Along-beam velocity data was calculated for a single upward-looking ADCP at a depth of 20 m,
523 with 30 bins, the first bin centred at 0.97 m from the transducer and with 0.1 m vertical bin centre
524 spacing. Velocities were calculated at one second intervals for a five minute observation period.
525 Surface wave wavelengths varied between 50 and 300 m and amplitudes between 0 and 2 m. The
526 radian frequency was calculated from the dispersion relation $c^2 = \frac{g}{k} \tanh(kh)$ where c is the wave
527 phase speed.

528 The along-beam velocity data was processed to calculate the second-order structure function for
529 separation distances up to the specified r_{\max} using a central-difference scheme. A background ε
530 level was then added to the structure function so that the effectiveness of the modified method
531 in recovering turbulence levels in the presence of wave orbital motions could be assessed. The
532 imposed background ε level varied logarithmically with wave amplitude from 1×10^{-10} to $1 \times$
533 $10^{-9} \text{ W Kg}^{-1}$. The standard and modified methods were then used to calculate ε estimates for
534 each bin based on r_{\max} values between 1.0 and 3.0 m. An average ε estimate was calculated as the

geometric mean of the individual values for all bins for which the structure function was resolved
for all $r \leq r_{\max}$.

Figure A1 compares the results for the standard (a,c,e,g) and modified (b,d,f,h) methods based
on $r_{\max} = 2.0$ m for water depths of 150 m (a,b), 75 m (c,d), 50 m (e,f) and 25 m (g,h). Subplots
(a) and (b) represent deep water waves, with subplot (a) being comparable to subplot (a) of figure
2, although the wavelength range 50 to 300 m in figure A1 equates to a wider wave period range
of 5.7 to 13.9 s. The figure shows that for the standard method, the bias introduced by the vertical
gradient in the wave orbital speed overwhelms the imposed background ε , with the level of bias
for a given wavelength and amplitude increasing slightly in shallower water depths.

The results from the modified method demonstrate that the method is generally effective in
recovering the imposed background ε levels, the effectiveness increasing with increasing wave-
length. Reducing the water depth has only a minimal impact, with a slight improvement in effec-
tiveness as the depth is reduced.

For the shortest wavelengths and largest wave amplitudes, the modified method exhibits a neg-
ative bias, resulting in calculated ε estimates lower than the imposed background values. The
is due to the structure function regression against $r^{2/3}$ failing to separate the linear term used to
calculate ε from the $(r^{2/3})^3$ term associated with the wave orbital motion. Increasing the imposed
background level or increasing the depth of the observations reduces the effect, whilst increasing
 r_{\max} increases the effect. This effectively introduces an observation-depth dependent limit on the
method sensitivity in the presence of high frequency waves.

The results from the tests with synthetic data demonstrate that providing the wave-induced or-
bital motion conforms to the standard equations, reducing the overall water depth does not signif-
icantly compromise the effectiveness of the modified method in removing bias in ε estimates due
to the presence of surface waves.

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705 TABLE 1. Median, 10th and 90th percentile wind stress scaled ε estimates for the three observation depths and
706 for both the standard and modified methods.

Depth (m)	Standard Method			Modified Method		
	10%ile	50%ile	90%ile	10%ile	50%ile	90%ile
24.0	3.14	9.15	31.03	0.42	1.11	3.85
42.5	0.82	2.33	7.01	0.18	0.69	1.90
52.5	0.55	1.78	6.27	0.18	0.80	2.71

707 TABLE 2. Median, 10th and 90th percentile buoyancy flux scaled ε estimates for the three observation depths
708 and for both the standard and modified methods.

Depth (m)	Standard Method			Modified Method		
	10%ile	50%ile	90%ile	10%ile	50%ile	90%ile
24.0	4.13	21.15	90.33	0.29	1.36	7.38
42.5	1.00	3.14	12.94	0.08	0.79	3.40
52.5	0.56	2.21	11.83	0.07	0.85	4.67

709 TABLE 3. Wind stress forcing. Mean R_{adj}^2 quality of fit for D_{LL} versus $r^{2/3}$ regressions for separation ranges
710 up to the specified r_{max} for the three observation depths and for both the standard and modified methods.

Depth (m)	Standard method			Modified method		
	$r_{\text{max}} = 1 \text{ m}$	2 m	3 m	1 m	2 m	3 m
24.0	0.58	0.81	0.84	0.59	0.85	0.93
42.5	0.58	0.80	0.85	0.58	0.83	0.91
52.5	0.39	0.57	0.67	0.39	0.58	0.70

711 TABLE 4. convective forcing. Mean R_{adj}^2 quality of fit for D_{LL} versus $r^{2/3}$ regressions for separation ranges up
712 to the specified r_{max} for the three observation depths and for both the standard and modified methods.

Depth (m)	Standard method			Modified method		
	$r_{\text{max}} = 1 \text{ m}$	2 m	3 m	1 m	2 m	3 m
24.0	0.50	0.78	0.83	0.51	0.83	0.92
42.5	0.41	0.71	0.80	0.41	0.75	0.85
52.5	0.31	0.51	0.62	0.31	0.52	0.66

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- Fig. 1.** Schematic of wave orbital motion contribution to the second-order structure function, D_{LL} . Monochromatic, deep water surface waves of amplitude A , period T_p , radian frequency ω , and wavenumber k , drive irrotational circular motions with speed at depth z (zero at surface, positive up) given by $v_{\max}(z) = A\omega e^{kz}$. In the absence of any other motion, the ADCP only measures the along-beam component of the wave orbital motion, such that $u(z, t) = v_{\max}(z) \sin(\omega t)$, the velocities being in phase between bins whilst varying in magnitude with bin depth. The turbulent velocity, $u' = u - \langle u \rangle$, retains the wave orbital motion since the bin mean over a sampling period, $\langle u \rangle_{T \gg T_p} \approx 0$. The second-order structure function is the mean of the turbulent velocity variance, $\langle (\delta u')^2 \rangle$, for a range of separation distances, see equation (2). In the presence of an along-beam gradient in wave orbital motion speed, $\langle (\delta u')^2 \rangle > 0$ for all separation distances, resulting in an unavoidable non-turbulent contribution to D_{LL} 42
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- Fig. 4.** Observed difference in rms vertical velocity, δw_{rms} , versus the difference in theoretical maximum wave orbital velocity magnitude, δv_{\max} for the three instruments with observations centred at depths 24.0 m (red), 42.5 m (orange) and 52.5 m (purple). Differences calculated over range $\delta z = 2.0$ m; δw_{rms} from earth coordinate transformed velocities with rms over 300 profiles per 5 minute sampling period; δv_{\max} based on monochromatic waves of amplitude half the observed significant wave height and with the observed average period. 46
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757 **Fig. A1.** Contour plots of $\log_{10}(\varepsilon)$ estimates from wave orbital velocities synthesized using general
 758 wave velocity equations (A1) for water depths (a,b) 150 m, (c,d) 75 m, (e,f) 50 m and (g,h)
 759 25 m, calculated using (a,c,e,g) standard and (b,d,f,h) modified structure function method.
 760 ADCP at 20 m depth, upward-looking with 30 bins with a vertical bin size of 0.1 m and the
 761 first bin centred at 0.97 m from the ADCP. Wave orbital velocities resolved at 1 s intervals
 762 for 300 s. A background ε level is imposed, varying with surface wave amplitude from
 763 $1 \times 10^{-10} \text{ W kg}^{-1}$ for amplitude 0 m to $1 \times 10^{-9} \text{ W kg}^{-1}$ for amplitude 2 m waves, such
 764 that in the absence of any wave-related bias, contours -9.1, -9.2 ... -9.9 would be equally
 765 spaced horizontal lines. 51

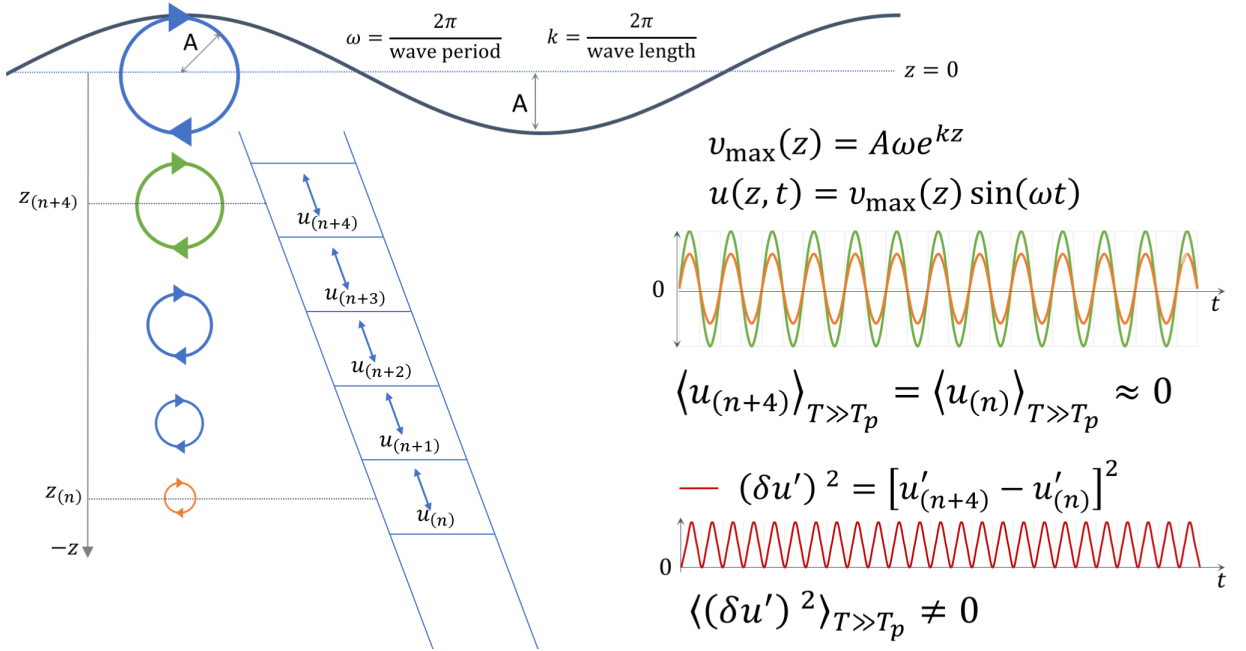
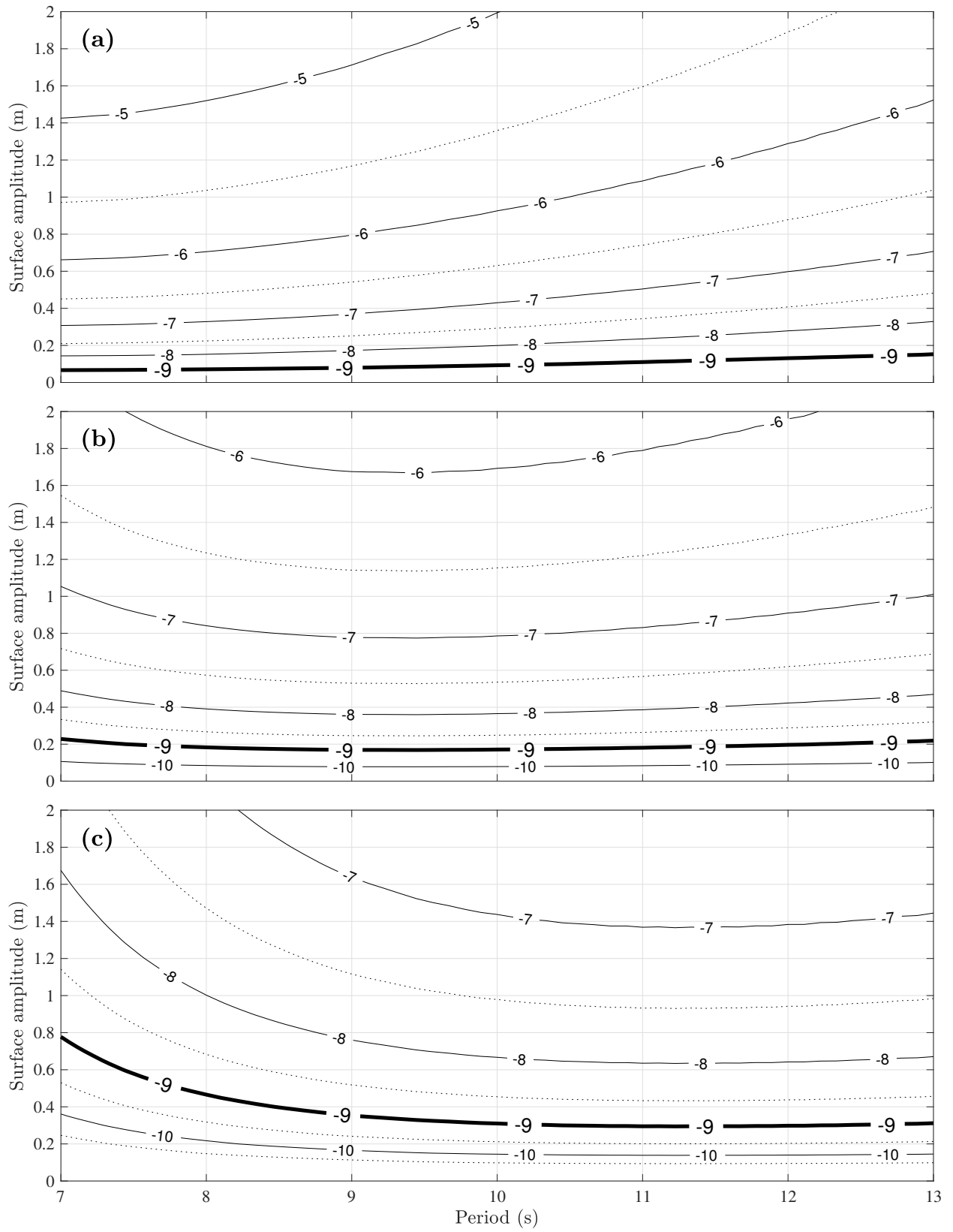
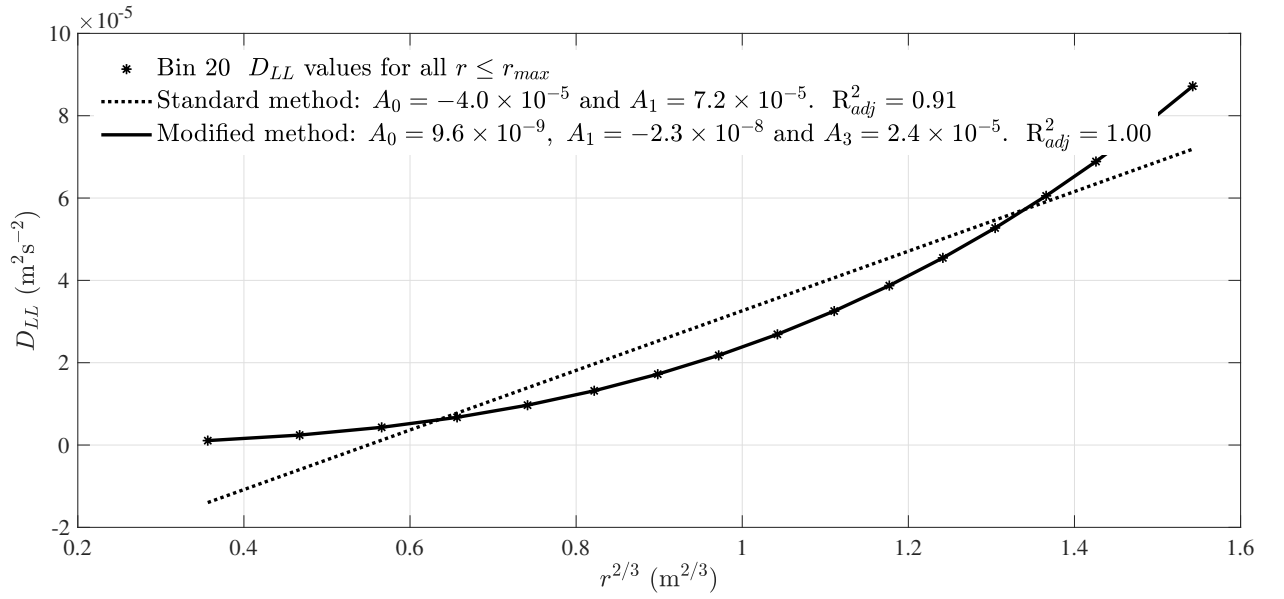


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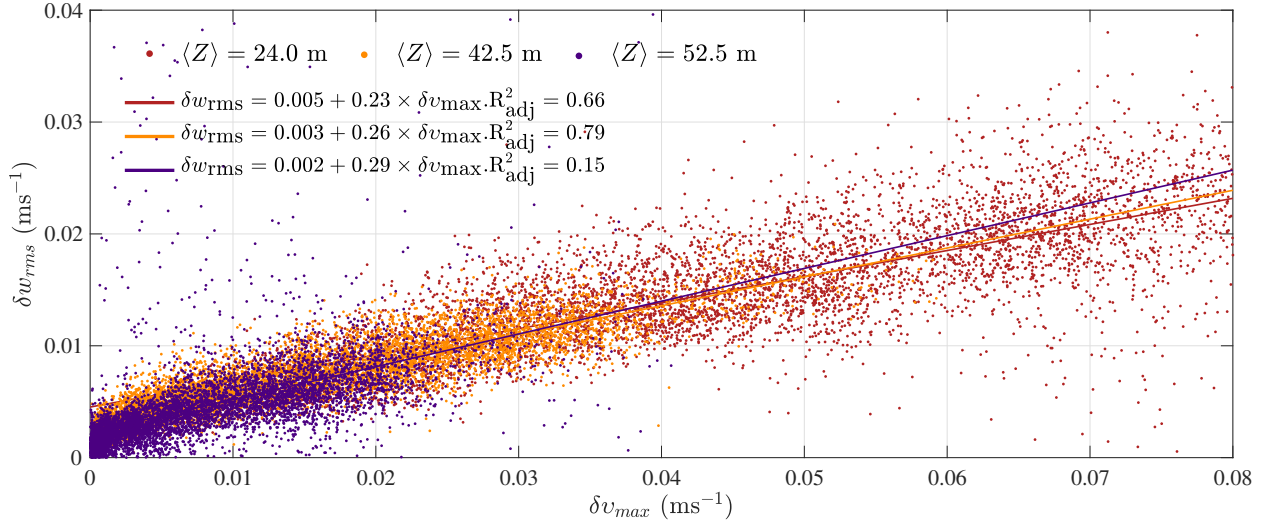


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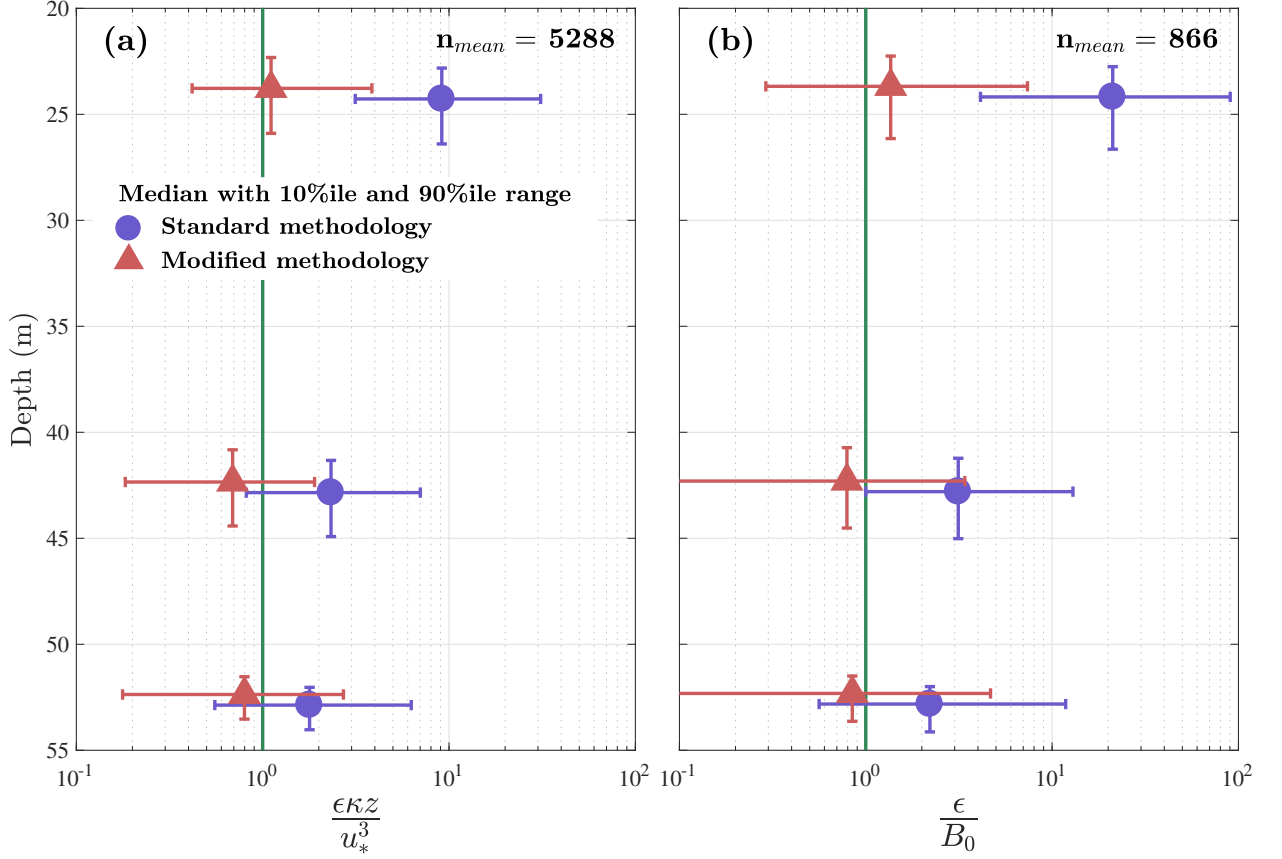


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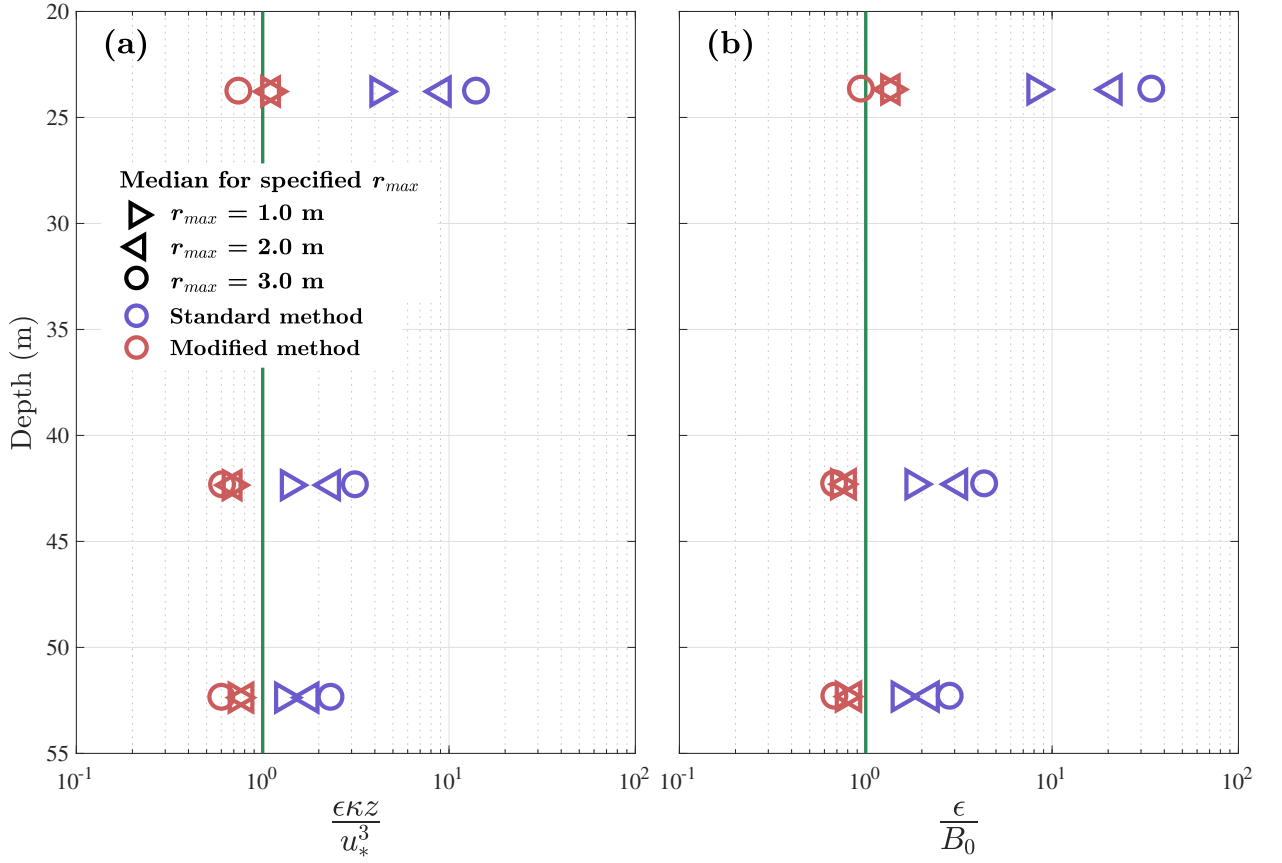


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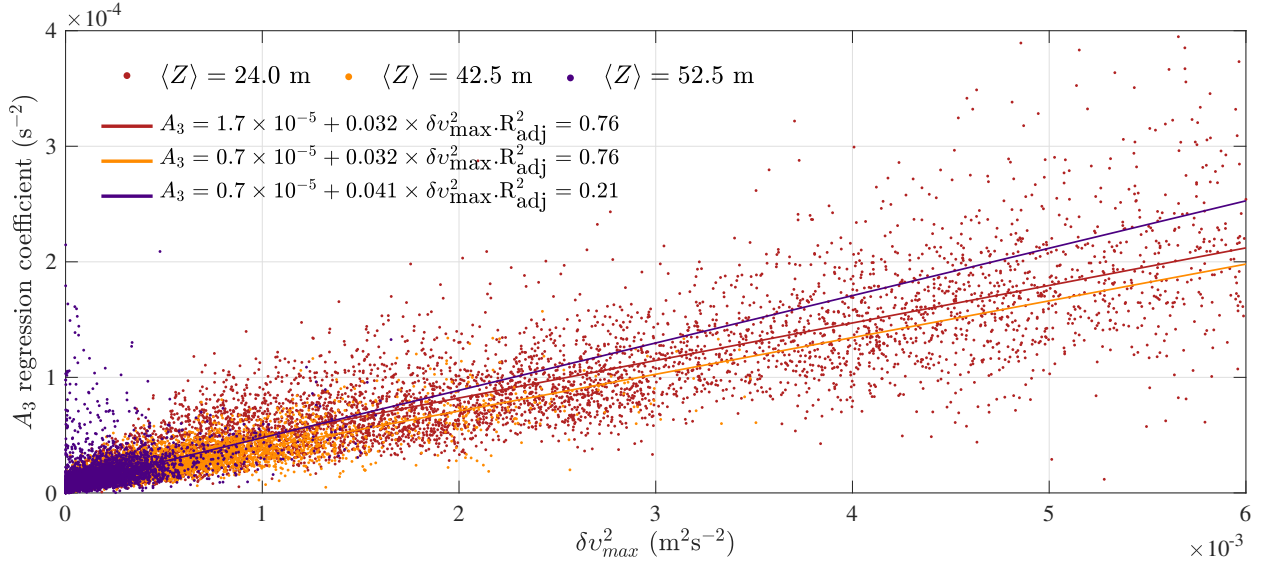
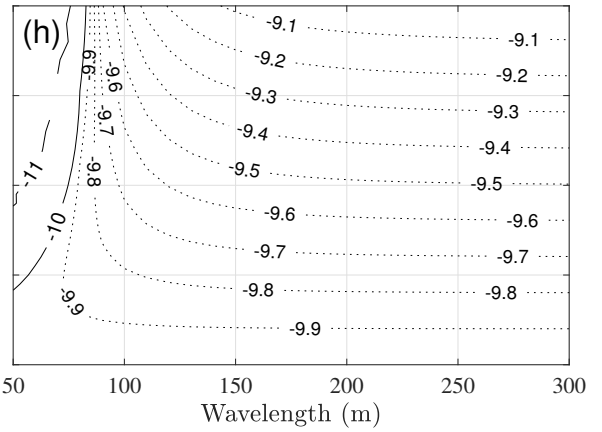
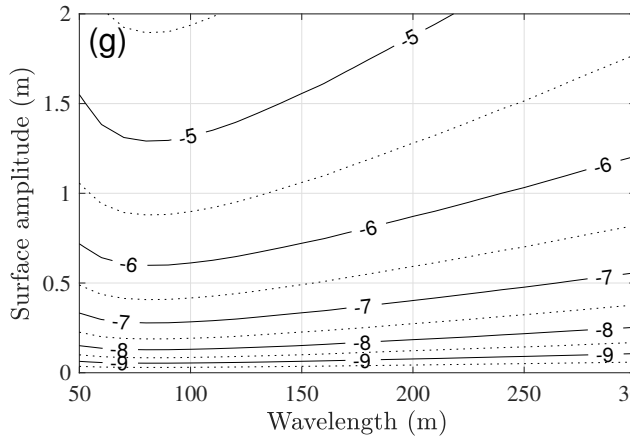
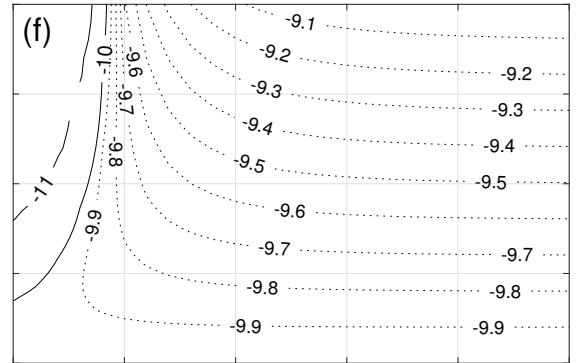
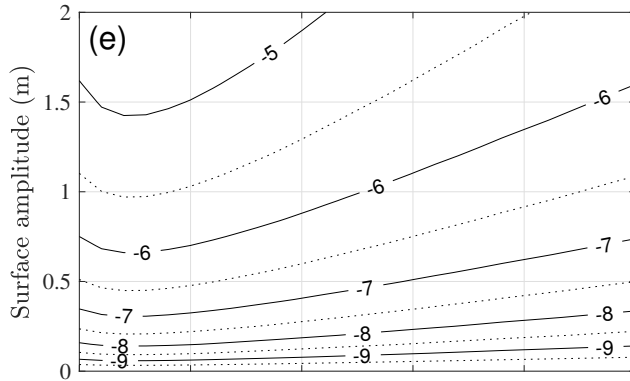
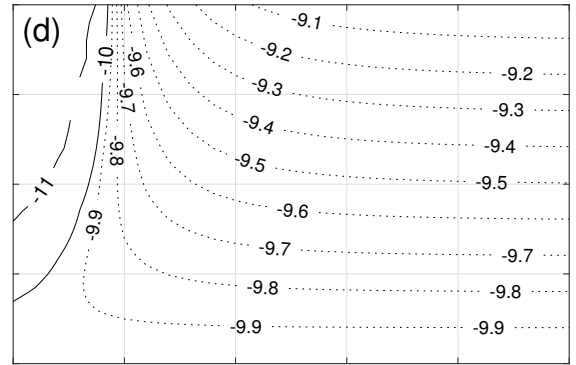
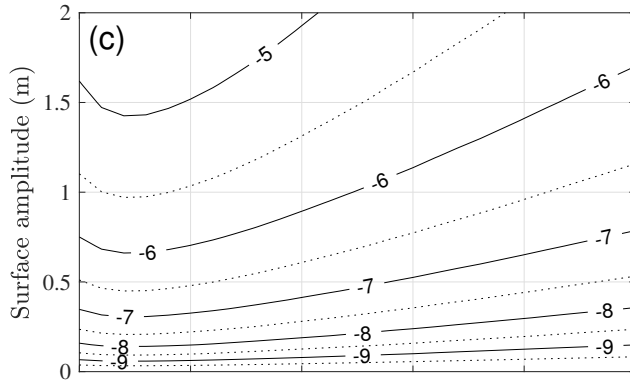
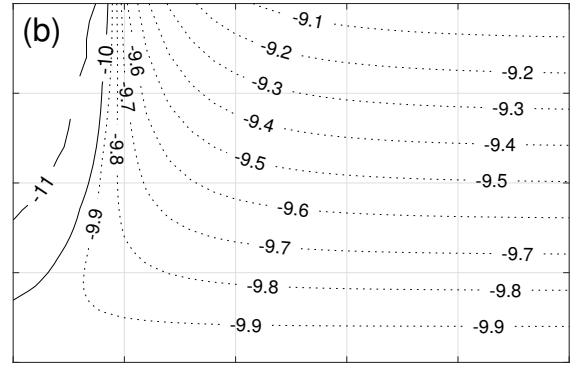
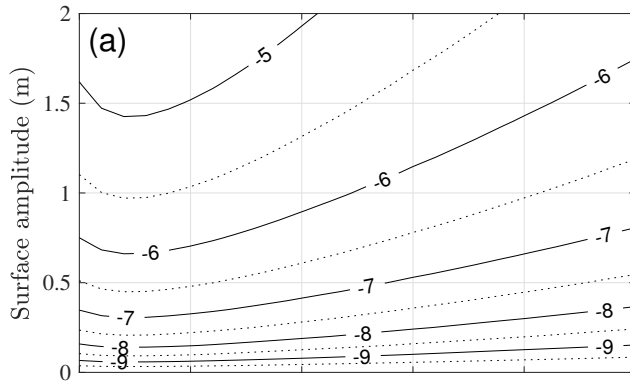


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